Influence of the nodal distribution on Element-Free Galerkin accuracy in a topology optimization context

Johannes T.B. Overvelde[‡*], Matthijs Langelaar[‡] and Fred van Keulen[‡]

[‡]Delft University of Technology
Mekelweg 2, 2828 CD Delft, the Netherlands
j.t.b.overvelde@student.tudelft.nl / m.langelaar@tudelft.nl / a.vankeulen@tudelft.nl

Key words: Element-free Galerkin method, meshless methods, topology and shape optimization.

ABSTRACT

In the field of structural topology optimization, the aim is to find the topology and shape of the optimal structure for a given problem. Various techniques have been developed, and most popular are the so-called density-based and level-set-based approaches [1]. Most of these methods map a material distribution onto a fixed grid of finite elements to analyze structural performance, where the structure is embedded in a larger computational mesh. In our research, we aim to explore the potential of meshless methods for use in topology optimization. Expected advantages are easier local refinement, smoothness of the solution and robustness in large deformations. Several studies have been published on topology optimization using meshfree methods as a direct replacement of finite elements, in which the nodal distribution remained unchanged during the optimization process, e.g. [2].

In contrast to earlier work, our investigation focuses on the possibility of exploiting the flexibility of the nodal distribution of a meshless method itself in an optimization setting. For example, we envision that by moving around the computational nodes in a design domain during the optimization process, computation effort could be concentrated to the regions where it is needed, i.e. the material regions. A key aspect here is the effect of the nodal distribution on the numerical solution, therefore we have performed an investigation of precisely this effect, and we here report on our initial findings.

We focus on the Element-free Galerkin (EFG) method [3]. The EFG method does not depend on elements to discretize the governing equations and is therefore more suited for problems with varying nodal distributions. However, in literature the EFG method is mostly applied to regular shaped domains with regular nodal distributions. We first investigate the effect of varying nodal distributions on the analysis accuracy, followed by a study to the influence of relative position and orientation of an embedded material domain to a larger computational grid, as typically used in topology optimization. In both studies a two-dimensional cantilever beam is used as a model problem, for which the EFG results can be compared to an analytical solution [3].

In the first study, the material domain remains unchanged and is discretized with random nodal distributions of 100 nodes. For the integration a background mesh is fitted to the material domain. A sample of the nodal distribution is shown in Fig. 1a. From these random nodal distributions a global effect on the accuracy of the EFG method can be determined. The global behavior of the structure can be expressed as the compliance ($C = \mathbf{u}^T\mathbf{f}$, with $\mathbf{u}$ and $\mathbf{f}$ vectors of displacements and applied loads, respectively), which is often used as objective function in topology optimization. For 5000 random nodal distributions the compliance is shown in Fig. 2a, normalized to the analytical value. The EFG solution proves highly sensitive to the nodal distribution. In contrast, a regular nodal distribution with 100 nodes closely matches the analytical result (compliance difference below 0.1%).

Secondly, we examine the influence of the relative position and orientation of a material domain, defined by an associated regular nodal distribution, to the background grid. Fig. 1b shows again the

cantilever beam modeled with a regular 100-nodal distribution. Here the integration domain extends beyond the boundary formed by the EFG nodes (hereafter: nodal boundary). The influence domains of nodes near the nodal boundary extend to integration points outside the nodal boundary. When the regular nodal grid is translated (by dx, dy) or rotated (by β), different integration points are included, resulting in variations in compliance, shown in Fig. 2b. Discontinuities occur when integration points enter or leave the nodal influence domains. Moreover, for some specific values of the angle, the compliance lowers drastically (more than a factor of 5). For these angles artificial stiffening occurs.

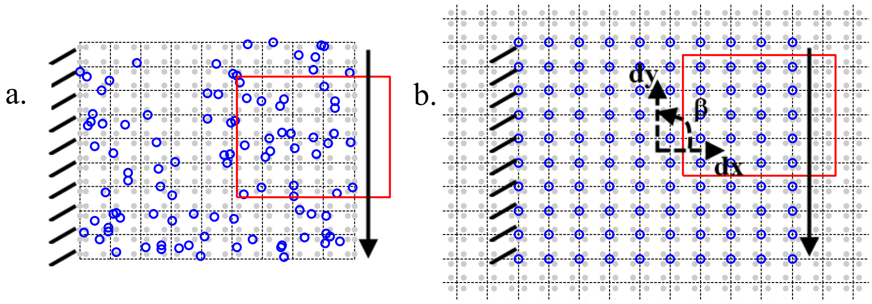

Figure 1: Nodal distributions used to obtain the results from Fig. 2. Integrals are evaluated numerically with Gauss integration. Integration points (grey dots) are distributed in a cell structure. The linear elasticity equations are discretized using the nodes (blue circles), of which one nodal influence domain is shown (red line).

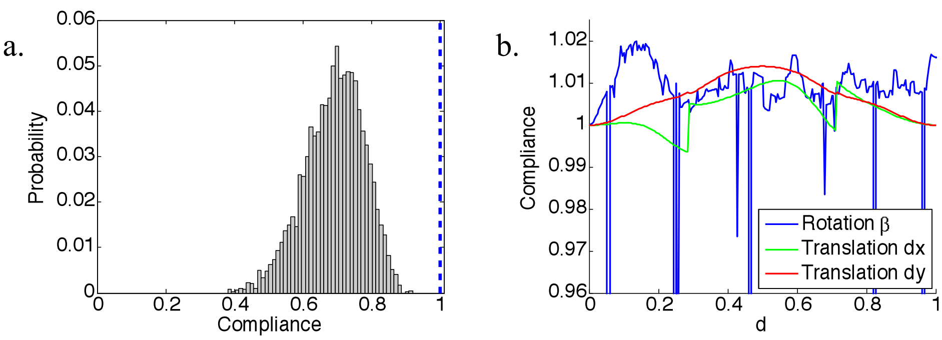

Figure 2: a) Normalized compliance for 5000 random nodal distributions, as in Fig. 1a. The blue dashed line represents a regular distribution. b) Compliance versus translations and rotations of the background mesh shown in Fig.1b. The size of the translations is normalized to one cell size and the rotation is normalized to pi/4.

In conclusion, there is a noticeable dependence of the EFG solution on the nodal distribution. Embedding the nodes in a larger domain can introduce substantial deviations of the solution. To the authors' knowledge, these phenomena of the EFG method have not been reported before. Caution is warranted regarding the suitability of the EFG formulation for topology optimization procedures involving moving nodes. Beyond the intended topology optimization application, our findings illustrate the importance of the nodal distribution used in EFG. Therefore, some systematic node distribution process may be required in other applications as well.

References

[1] M.P. Bendsøe and O. Sigmund: Topology optimization: theory, methods, and applications. Springer, 2003.

[2] J. Zheng, S. Long, G. Li: The topology optimization design for continuum structures based on the element free Galerkin method. *Eng. Analysis with Boundary Elements*, 34 (2010), 666-672.

[3] T. Belytschko, Y.Y. Lu and L. Gu: Element-free Galerkin methods. *International Journal for Numerical Methods in Engineering*, 37 (1994), 229-256.